WNE Linear Algebra Final Exam

Series A

1 February 2012

Please use separate sheets for different problems. Please provide the following data on each sheet

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem and its series.**

Problem 1.
Let $V = \text{lin}((1,2,1,1),(1,0,0,2),(1,4,2,0),(3,2,1,5))$ be a subspace of $\mathbb{R}^4$.

a) find basis and dimension of the space V,
b) find a system of linear equations which set of solutions is equal to V,

Problem 2.
Let $W \subset \mathbb{R}^5$ be a subspace given by the homogeneous system of linear equations

$$\begin{cases} x_1 + x_2 + 2x_3 + 2x_4 - x_5 = 0 \\ 2x_1 + 3x_2 + x_3 - x_4 + x_5 = 0 \end{cases}$$

a) find basis and dimension of the space W,
b) let $V_s = \{(x_1, x_2, x_3, x_4, x_5) \colon x_2 - 3x_3 + sx_4 + 3x_5 = 0\}$ for $s \in \mathbb{R}$. Give all $s \in \mathbb{R}$ for which $W \subset V_s$

Problem 3.
Let endomorphism $\varphi_t \colon \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be given by the formula $\varphi_t((x_1, x_2, x_3)) = (2x_1 + 2x_2 + tx_3, 2x_1 + 5x_2 + 6x_3, x_3)$.

a) find eigenvalues of φ_t and bases of the corresponding eigenspaces for $t = 0$,
b) for which $t \in \mathbb{R}$ there exists a basis of $\mathbb{R}^3$ such that the matrix of φ_t relative to it is diagonal.

Problem 4.
Let $\mathcal{A} = ((1,0,1),(0,1,0),(1,0,2))$ be an ordered basis of $\mathbb{R}^3$ and let $\mathcal{B} = ((1,1),(1,2))$ be and ordered basis of $\mathbb{R}^2$. The linear transformation $\psi \colon \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ is given by the matrix $M(\psi)_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$. The linear transformation $\varphi \colon \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is given by the formula $\varphi((x_1, x_2)) = (x_1 - x_2, x_1 + x_2)$.

a) find formula of ψ,
b) compute matrix $M(\varphi \circ \psi)_{\mathcal{A}}^{\mathcal{B}}$.

Problem 5.
Let $V = \text{lin}((1,2,0,1),(2,3,0,2),(0,0,1,0))$ be subspace of $\mathbb{R}^4$ and let $P = (1,1,1,2), Q = (3,0,0,1)$ be points in $\mathbb{R}^4$.

a) find equation of the affine space $H = P + V \subset \mathbb{R}^4$,

b) find a parametrization of the line containing Q and perpendicular to H, find the image of Q under the orthogonal projection onto H.

Problem 6.

Let $A_t = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 3 & t \\ 2 & 3 & 1 \end{bmatrix}$ for $t \in \mathbb{R}$.

a) for which $t \in \mathbb{R}$ the matrix A_t is invertible?
b) for which $t \in \mathbb{R}$ the entry in the second column and the first row of A^{-1} is equal to -2?

Problem 7.

Let $q_t \colon \mathbb{R}^3 \longrightarrow \mathbb{R}$ be a quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 4x_1x_2 + 2tx_2x_3$.

a) for which $t \in \mathbb{R}$ the form q_t is positive definite?
b) check if q_t is either positive semidefinite or negative semidefinite for $t = 1$.

Problem 8.

Consider the following linear programming problem $x_1 - x_2 - x_5 \rightarrow \min$ in the standard form with constraints

$$\begin{cases} x_1 - x_2 + 2x_3 + 2x_5 = 5 \\ x_2 + 2x_3 + x_4 + x_5 = 2 \end{cases} \text{ and } x_i \geqslant 0 \text{ for } i = 1, \ldots, 5$$

a) which of the basic solutions $\mathcal{B}_1 = \{1, 4\}, \mathcal{B}_2 = \{3, 4\}$ are feasible?
b) solve the above linear programming problem using simplex method.